Freedom Valley School, Bardoli

Class:12th **WORKSHEET** **Chapter no:** 5

Teacher's sign: **Sub**: MATHS **Principal sign:**

CH-5 **Continuity And Differentiability**

1. LIMITS AND CONTINUITY OF A FUNCTION

Objective Qs (1 mark)

1. If $f(x) = \begin{cases} \frac{kx}{|x|}, \text{ if } x < 0 \\ 3, \text{ if } x \geq 0 \end{cases}$ is continuous at $x = 0$, then the value of k is:

(a) -3
(b) 0
(c) 3
(d) any real number

[CBSE SQP 2023]

2. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function, is continuous at:
(a) 4
(b) 1.5
(c) 1
(d) -2

[CBSE Term-1 2021]

3. The value of $k(k < 0)$ for which the function f defined as:

$$f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x} & , x \neq 0 \\ \frac{1}{2} & , x = 0 \end{cases}$$

is continuous at $x = 0$ is:
(a) ± 1
(b) -1
(c) $\pm \frac{1}{2}$
(d) $\frac{1}{2}$

[CBSE Term-1 SQP 2021]

4. The point(s), at which the function f given by $f(x) = \begin{cases} \frac{x}{|x|}, x < 0 \\ -1, x \geq 0 \end{cases}$ is continuous, is/are:

(a) $x \in R$
(b) $x = 0$
(c) $x \in \mathrm{R} - \{0\}$
(d) $x = -1$ and 1

[CBSE Term-1 SQP 2021]

5. $f(x)=\begin{cases}3x-8 & \text{if } x\leq 5\\ 2k & \text{if } x>5\end{cases}$ is continuous, find k.

(a) $\frac{2}{7}$

(b) $\frac{3}{7}$

(c) $\frac{4}{7}$

(d) $\frac{7}{2}$

[CBSE Term-1 2021]

Very Short & Short Qs (1 - 3 marks)

6. Find the value of λ so that the function f defined by

$$f(x)=\begin{cases}\lambda x, & \text{if } x\leq \pi\\ \cos x, & \text{if } x>\pi\end{cases}$$

is continuous at $x=\pi$.

[CBSE 2020]

7. Determine the value of ' k ' for which the following function is continuous at $x=3$.

$$f(x)=\begin{cases}\dfrac{(x+3)^2-36}{x-3}, & x\neq 3\\ k & ,x=3\end{cases}$$

[CBSE 2017]

8. Find the values of p and q, for which

$$f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x} & ,\text{ if } x<\dfrac{\pi}{2}\\ p & ,\text{ if } x=\dfrac{\pi}{2}\\ \dfrac{q(1-\sin x)}{(\pi-2x)^2} & ,\text{ if } x>\dfrac{\pi}{2}\end{cases}$$

is continuous at $x=\frac{\pi}{2}$.

[CBSE 2016]

9. Find the value of the constant k so that the function f, defined below, is continuous at $x=0$, where

$$f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2} & ,\text{ if } x\neq 0\\ k & ,\text{ if } x=0\end{cases}$$

[CBSE SQP 2014]

Long Qs (4 - 5 marks)

10.

$$\text{If } f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x} & x < 0 \\ \dfrac{2}{\dfrac{\sqrt{1+bx}-1}{x}} & x > 0 \end{cases} \text{ is}$$

continuous at $x = 0$ then find the values of a and b.

[CBSE 2016]

2. DIFFERENTIABILITY

Objective Qs (1 mark)

11. The set of all points where the function $f(x) = x + |x|$ is differentiable, is:
(a) $(0, \infty)$
(b) $(-\infty, 0)$
(c) $(-\infty, 0) \cup (0, \infty)$
(d) $(-\infty, \infty)$

[CBSE SQP 2023]

12. The function $f(x) = x|x|$ is:

(a) continuous-and differentiable at $x = 0$.

(b) continuous but not differentiable at $x = 0$.

(c) differentiable but not continuous at $x = 0$.

(d) neither differentiable nor continuous at $x = 0$.

[CBSE 2023]

13. The function given below at $x = 4$ is:

$$f(x) = \begin{cases} 2x + 3, x \le 4 \\ x^2 - 5, x > 4 \end{cases}$$

(a) continuous but not differentiable

(b) differentiable but not continuous

(c) continuous as well as differentiable

(d) neither continuous nor differentiable

[Delhi. Gov. Term-1 SQP 2021]

14. The function $f(x) = \begin{cases} x^2 & \text{for} \quad x \leq 1 \\ 2-x & \text{for} \quad x \geq 1 \end{cases}$ is:

(a) Not differentiable at $x = 1$

(b) Differentiable at $x = 1$

(c) Not continuous at $x = 1$

(d) Neither continuous nor differentiable at $x = 1$

[CBSE Term-1 2021]

Very Short & Short Qs (1-3 marks)

15. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1$.

[CBSE 2020]

16. Let, $f(x) = x|x|$, for all $x \in R$. Check its differentiability at $x = 0$.

[CBSE 2020]

17. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x = 1$

[CBSE 2020]

18. Find the values of a and b, if the function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a & , x \leq 1 \\ bx + 2 & , x > 1 \end{cases}$$

is differentiable at $x = 1$.

[CBSE 2016]

19. For what value of λ, the function defined by $f(x) = \begin{cases} \lambda(x^2+2), \text{ if } x \leq 0 \\ 4x + 6, x > 0 \end{cases}$ is continuous at $x = 0$? Hence, check the differentiability of $f(x)$ at $x = 0$.

[CBSE 2015]

20. Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 < x \leq 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

[CBSE 2015]

3. DERIVATIVES

Objective Qs (1 mark)

21. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to:
(a) $\frac{-y}{x}$
(b) $\frac{y}{x}$
(c) $\sec^2\left(\frac{y}{x}\right)$
(d) $-\sec^2\left(\frac{y}{x}\right)$

[CBSE 2023]

22. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is:
(a) e^{y-x}
(b) e^{x+y}
(c) $-e^{y-x}$
(d) $2e^{x-y}$

[Delhi Gov. SQP 2022, CBSE Term-1 SQP 2021]

23. The derivative of $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ w.r.t. $\sin^{-1} x, \frac{1}{\sqrt{2}} < x < 1$, is:
(a) 2
(b) $\frac{\pi}{2} - 2$
(c) $\frac{\pi}{2}$
(d) -2

[CBSE Term-1 SQP 2021]

24. If $(x^2+y^2)^2 = xy$, then $\frac{dy}{dx}$ is:
(a) $\frac{x+4x(x^2+y^2)}{4y(x^2+y^2)-x}$
(b) $\frac{y-4x(x^2+y^2)}{x+4(x^2+y^2)}$
(c) $\frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$
(d) $\frac{4y(x^2+y^2)-x}{y-4x(x^2+y^2)}$

[CBSE Term- 1 2021]

25. If $y = \log(\cos e^x)$, then $\frac{dy}{dx}$ is:
(a) $\cos e^{x-1}$
(c) $e^x \sin e^x$
(b) $\mathrm{e}^{-x} \cos e^x$
(d) $-e^x \tan e^x$

[CBSE Term-1 SQP 2021]

26. If $y^2(2-x)=x^3$, then $\left(\frac{dy}{dx}\right)_{(1,1)}$ is equal to:

(a) 2
(b) -2
(c) 3
(d) $-\frac{3}{2}$

[CBSE Term-1 2021]

Very Short & Short Qs (1 - 3 marks)

27. If $f(x)=\begin{cases} ax+b; & 0<x\leq 1 \\ 2x^2-x; & 1<x<2 \end{cases}$ is a differentiable function in (0,2), then find the values of a and b.

[CBSE 2023]

28. If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$, then prove that $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$.

[CBSE SQP 2022]

29. If $y=\tan^{-1}\ x+\cot^{-1}\ x, x\in R$, then find $\frac{dy}{dx}$.

[CBSE 2020]

30. If $\cos\,(xy)=k$, where k is a constant and $xy\neq n\pi, n\in Z$, then $\frac{dy}{dx}$ is equal to ....

[CBSE 2020]

31. If $x=a\sec\ \theta, y=b\tan\theta$, then find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$.

[CBSE 2020]

32. If $y=e^{x^2\cos\ x}+(\cos\ x)^x$, then find $\frac{dy}{dx}$.

[CBSE 2020]

33. Differentiate $\tan^{-1}\left(\frac{\cos\ x-\sin\ x}{\cos\ x+\sin\ x}\right)$ with respect to x.

[CBSE 2018]

34. If $x=a(2\theta-\sin\ 2\theta)$ and $y=a(1-\cos\ 2\theta)$, find $\frac{dy}{dx}$ when $\theta=\frac{\pi}{3}$.

[CBSE 2018]

35. Find $\frac{dy}{dx}$ at $x=1, y=\frac{\pi}{4}$, if $\sin^2\ y+\cos\ xy=k$.

[CBSE 2017]

36. If $y=\sin^{-1}\left(6x\sqrt{1-9x^2}\right), -\frac{1}{3\sqrt{2}}<x<\frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

[CBSE 2017]

37. If $y = \tan^{-1}\left(\frac{a}{x}\right) + \log\sqrt{\frac{(x-a)}{(x+a)}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{(x^4-a^4)}$

[CBSE 2014]

Long Qs

38. If $\log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$ Show that $\frac{dy}{dx}$

$$= \frac{x+y}{x-y}.$$

[CBSE 2019]

39. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then show that

$$y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$ [CBSE 2015]

40. If $y = \frac{x\cos^{-1}x}{\sqrt{(1-x^2)}} - \log\sqrt{(1-x^2)}$, then prove that $\frac{dy}{dx} = \cos^{-1}\frac{x}{(1-x^2)^{3/2}}$.

[CBSE 2015]

41. If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, show that at $t = \frac{\pi}{4}, \left(\frac{dy}{dx}\right) = \frac{b}{a}$.

[CBSE 2015]

42. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, then prove that : $\frac{dy}{dx} = \frac{(x+y)}{(x-y)}$.

[CBSE 2015]

43. If $e^x + e^y = e^{x+y}$, then show that $\frac{dy}{dx} = -e^{y-x}$.

[CBSE 2014]

44. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

[CBSE 2014]

45. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{x}{4}$, if $x = ae^\theta(\sin\theta - \cos\theta)$ and $y = ae^\theta(\sin\theta + \cos\theta)$.

[CBSE 2014]

4. SECOND ORDER DERIVATIVE

46. If $y = 5\cos x - 3\sin x$, then $\frac{d^2y}{dx^2}$ is equal to:
(a) $-y$
(b) y
(c) $25y$
(d) $9y$

[CBSE Term-1 SQP 2021]

47. If $x = a\sec\ \theta, y = b\tan\ \theta$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is:

(a) $\frac{-3\sqrt{3}b}{a^2}$

(b) $\frac{-2\sqrt{3}b}{a}$

(c) $\frac{-3\sqrt{3}b}{a}$

(d) $\frac{-b}{3\sqrt{3}a^2}$

[CBSE Term-1 SQP 2021]

48. If $y = \log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ is equal to:

(a) $\frac{-1}{x}$

(b) $-\frac{1}{x^2}$

(c) $\frac{2}{x^2}$

(d) $-\frac{2}{x^2}$ [CBSE 2020]

Very Short & Short Qs (1 - 3 marks)

49. If $y = \sqrt{ax + b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$.

[CBSE 2023]

50. If $x = a\cos\ \theta, y = b\sin\ \theta$, then find $\frac{d^2y}{dx^2}$.

[CBSE 2020]

51. If $x = at^2, y = 2at$ then find $\frac{d^2y}{dx^2}$.

[CBSE 2020]

52. If $e^y(x + 1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

[CBSE 2017]

53. If $x^m \cdot y^n = (x + y)^{m+n}$, prove that $\frac{d^2y}{dx^2} = 0$.

[CBSE 2017]

54. If $y = 2\cos\ (\log\ x) + 3\sin\ (\log\ x)$, prove that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

[CBSE 2016]

55. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$.

[CBSE 2015, 2014]

Long Qs (4 - 5 marks)

56. If $x = \sin\ t$ and $y = \sin\ pt$, prove that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0 \text{ [CBSE 2019]}$$

57. If $y = (\sin^{-1}\ x)^2$, then prove that:

$$(1 - x^2)\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right) - 2 = 0$$

[CBSE 2019]

58. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

[CBSE 2016, 2014]

59. If $y = 2\cos\ (\log\ x) + 3\sin\ (\log\ x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

[CBSE 2016]

60. If $x = a\left(\cos\ t + \log\ \tan\ \frac{t}{2}\right)$ and $y = a\sin\ t$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$.

[CBSE 2014]

Freedom Valley School, Bardoli

Worksheet

Class: 12th Sub: Mathematics

Date: 23/07/2025 Principal sign :

Chap 6 -Application of Derivatives

1. RATE OF CHANGE OF QUANTITIES

Case Based Qs (4- 5 marks)

1. The relation between the height of the plant (' y ' in cm) with respect to its exposure to the sunlight is governed by the following equation $y=4x-\frac{1}{2}x^2$, where ' x ' is the number of days exposed to the sunlight, for $x \leq 3$.

Based on the above information, answer the following questions:

(A) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(B) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?

[CBSE SQP 2023]

Very Short & Short Qs $(1-3$ **marks)**

2. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.

[CBSE 2023]

3. A man 1.6 m tall walks at the rate of $0.3\ \mathrm{m/s}$ away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?

[CBSE SQP 2022]

4. The radius of a circle is increasing at the uniform rate of $3\ \mathrm{cm/sec}$. At the instant when the radius of the circle is 2 cm, find its area increases at the rate of $\mathrm{cm}^2/\mathrm{sec}$.

[CBSE 2020]

5. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

[CBSE 2019]

6. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of its edge is 12 cm ?

[CBSE 2019]

7. The total cost $C(x)$ in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

[CBSE 2018]

8. The length x of a rectangle is decreasing at the rate of 5 cm/ minute and the width y is increasing at the rate of 4 cm/ minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of:

(A) the perimeter.

(B) area of rectangle.

[CBSE 2017]

9. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

[CBSE 2017]

Long Qs (4 - 5 marks)

10. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing, when the side of the triangle is 20 cm.

[CBSE 2015]

2. INCREASING AND DECREASING FUNCTIONS

Objective Qs (1 mark)

11. The real function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is:

(a) strictly increasing in $(-\infty, -2)$ and strictly decreasing in $(-2, \infty)$.

(b) strictly decreasing in $(-2,3)$.

(c) strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3, \infty)$.

(d) strictly decreasing in $(-\infty, -2) \cup (3, \infty)$.

[CBSE Term-1 SQP 2021]

12. The value of b for which the function $f(x) = x + \cos x + b$ is strictly decreasing over R is :
(a) $b < 1$
(b) no value of b exists
(c) $b \leq 1$
(d) $b \geq 1$

[CBSE Term-1 SQP 2021]

13. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing.
(a) $(-\infty, 2) \cup (2, \infty)$
(b) $(2, \infty)$
(c) $(-\infty, 2)$
(d) $(-\infty, 2] \cup (2, \infty)$

[CBSE Term-1 SQP 2021]

14. The interval in which $y = x^2 e^{-x}$ is increasing, is:
(a) $(-\infty, \infty)$
(b) $(-2,0)$
(c) $(2, \infty)$
(d) $(0,2)$

[CBSE Term-1 2021]

Case Based Qs (4 **- 5 marks)**

15. The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V.

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and $t = 1,2,3$ ... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions:

(A) Can the above function be used to estimate number of vehicles in the year 2000? Justify.

(B) Prove that the function $V(t)$ is an increasing function.

[CBSE 2023]

16. The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \le x \le 12$, m being a constant, where $f(x)$ is the temperature in $^\circ$F at x days.

Based on the above information, answer the following questions:

(A) Is the function differentiable in the interval $(0,12)$? Justify your answer.

(B) If 6 is the critical point of the function, then find the value of the constant m.

(C) Find the intervals in which the function is strictly increasing/strictly decreasing.

[CBSE SQP 2022]

Very Short & Short Qs $(1-3$ **marks)**

17. Find the interval/s in which the function $f: R \rightarrow R$ defined by $f(x) = xe^x$, is increasing.

[CBSE SQP 2023]

18. Check whether the function $f: R \rightarrow R$ defined by $f(x) = x^3 + x$, has any critical point/s or not? If yes, then find the point/s.

[CBSE SQP 2023]

19. Find the interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing.

[CBSE 2020]

20. Find the intervals in which the function f given by $f(x) = \tan x - 4x, x \in \left(0, \frac{\pi}{2}\right)$ is :

(A) Strictly increasing

(B) Strictly decreasing

[CBSE 2020]

21. Find the interval in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is:

(A) strictly increasing.

(B) strictly decreasing.

[CBSE 2018]

22. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

[CBSE 2017]

23. Find the interval in which $f(x) = \sin\ 3x - \cos\ 3x, 0 < x < \pi$, is strictly increasing or strictly decreasing.

[CBSE 2016]

24. Find the value (s) of x for which $y = [x(x-2)]^2$ is an increasing function.

[CBSE 2014]

25. Prove that the function f defined by $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in $(-1,1)$. Hence, find the intervals in which $f(x)$ is:

(A) strictly increasing.

(B) strictly decreasing.

[CBSE 2014]

26. Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is

(a) strictly increasing (b) strictly decreasing.

[CBSE 2014]

3. MAXIMA AND MINIMA

Objective Qs (1 mark)

27. The maximum value of $\left(\frac{1}{x}\right)^x$ is:
(a) $e^{1/e}$
(b) e
(c) $\left(\frac{1}{e}\right)^{1/e}$
(d) e^e

[CBSE Term-1 2021]

28. The maximum value of $[x(x-1)+1]^{1/3}$, $0 \leq x \leq 1$ is:
(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) $\left(\frac{1}{3}\right)^{1/3}$

[CBSE Term-1 SQP 2021]

29. The least value of the function $f(x) = 2\cos\ x + x$ in the closed interval $\left[0, \frac{\pi}{2}\right]$ is:

(a) 2

(b) $\frac{\pi}{6} + \sqrt{3}$

(c) $\frac{\pi}{2}$ (d) The least value does not exist.

[CBSE Term-1 SQP 2021]

30. The area of a trapezium is defined by function f and is given by

$$f(x) = (10 + x)\sqrt{100 - x^2}$$

Then the area when it is maximised is:
(a) $75\ \text{cm}^2$
(b) $7\sqrt{3}\ \text{cm}^2$
(c) $75\sqrt{3}\ \text{cm}^2$
(d) $5\ \text{cm}^2$

[CBSE Term-1 SQP 2021]

31. The absolute minimum value of the function $f(x) = x^3 - 12x$ on the interval [0,3] is:
(a) 0
(b) -9
(c) -16
(d) -19

[CBSE Term-1 2021]

32. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is:
(a) 15
(b) 12
(c) 9
(d) 0

[CBSE 2020]

33. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^3$, then
Assertion (A): $f(x)$ has a minimum at $x = 1$.

Reason (R): When $\frac{d}{dx}(f(x)) < 0, \forall x \in (a-h, a)$, and $\frac{d}{dx}(f(x)) > 0, \forall x \in (a, a+h)$; where ' h ' is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

[CBSE SQP 2023]

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

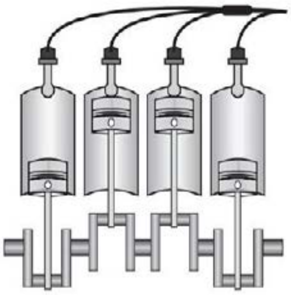
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

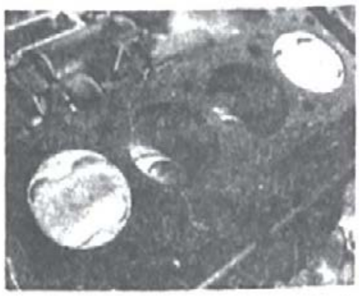
(c) (A) is true but (R) is false.

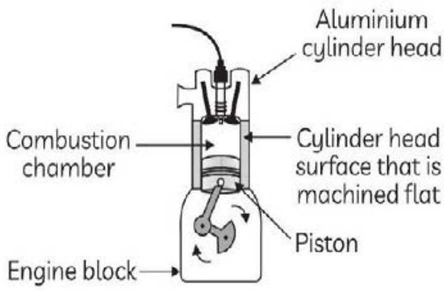
(d) (A) is false but (R) is true.

Case Based Qs (4 - 5 marks)

34. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.







The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi \text{cm}^2$.

Based on the above information, answer the following questions:

(A) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.
(B) Find $\frac{dV}{dr}$.

(C) Find the radius of cylinder when its volume is maximum.

OR

For maximum volume, $h > r$. State true or false and justify.

[CBSE 2023]

35. In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Based on the above information, answer the following questions:

(A) If the length and the breadth of the rectangular field be $2x$ and $2y$ respectively, then find the area function in terms of x.

(B) Find the critical point of the function.

(C) Use first derivative test to find the length $2x$ and width $2y$ of the soccer field (in terms of a and b) that maximize its area.

[CBSE SQP 2022]

Very Short & Short Qs (1 - 3 marks)

36. If $f(x)=\frac{1}{4x^2+2x+1}; x \in R$, then find the maximum value of $f(x)$. [CBSE SQP 2023]

37. Find the maximum profit that a company can make, if the profit function is given by $P(x)= 72+42x-x^2$, where x is the number of units and P is the profit in rupees.

[CBSE SQP 2023]

38. Find the absolute minimum volume of $f(x)=2\sin\ x$ in $\left(0,\frac{3\pi}{2}\right)$.

[CBSE 2020]

39. Find the least value of the function $f(x)=ax+\frac{b}{x},(a>0,b>0,x>0)$.

[CBSE 2020]
maximum light through the whole opening.

[CBSE 2018]

40. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone and greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

[CBSE 2020]

41. Find the minimum value of $(ax+by)$, where $xy=c^2$.

[CBSE 2020, 15]

42. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹70 per sq. meters for the base and ₹ 45 per square meter for sides. What is the cost of least expensive tank?

[CBSE 2019]

43. Find the point on the curve $y^2 = 4x$, which is nearest to the point $(2, -8)$.

[CBSE 2019]

44. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the volume of the largest cylinder inscribed in a sphere of radius R.

[CBSE 2019]

45. A window is of the form of a semi-circle with a rectangle on its diameter. The total perimeter of the window is 10 m. Find the dimensions of the window to admit

46. An open tank with a square base and vertical sides is to be construted from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width.

[CBSE 2018]

47. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1} \frac{1}{\sqrt{3}}$.

[CBSE 2016]

48. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

[CBSE 2016, 14]

49. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3r}$.

[CBSE 2016]

50. Find the local maxima and local minima of the function $f(x) = \sin x - \cos x, 0 < x < 2\pi$. Also find the local maximum and local minimum values.

[CBSE 2015]

51. The sum of the perimeters of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle.

[CBSE 2014]